PH16212, Homework 7

Deadline Dec. 23, 2019

1. (Lee-Pomeransky's master integral counting). In this exercise, we consider the Lee and Pomeransky's master integral counting algorithm. (`https://arxiv.org/pdf/1308.6676.pdf`) Note that this algorithm has counter examples.

- Consider the two-loop five-point planar "pentabox" integral:

$$\begin{gathered} D_1 = l_1^2, \quad D_2 = (l_1 - p_1)^2, \quad D_3 = (l_1 - p_1 - p_2)^2, \quad D_4 = (l_1 - p_1 - p_2 - p_3)^2 \\ D_5 = l_2^2, \quad D_6 = (l_2 - p_5)^2, \quad D_7 = (l_2 - p_4 - p_5)^2, \quad D_8 = (l_1 + l_2)^2 \\ D_9 = (l_1 + p_5)^2, \quad D_{10} = (l_2 + p_1)^2, \quad D_{11} = (l_2 + p_2)^2 \end{gathered} \tag{1}$$

  with $p_1^2 = p_2^2 = p_3^2 = p_4^2 = p_5^2 = 0$. Use either the Lee-Pameransky polynomial G or the Baikov polynomial P *on cut*, and run the critical point counting argument in *1308.6676* to find the number of master integrals on the sector $(1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0)$.

  Hint: it is not a bad idea to set $s_{12} = 1$, $s_{23} = 3$, $s_{34} = 13$, $s_{45} = 19$, $s_{51} = 31$ to speed up the computation. You can use the Groebner basis method for the critical point counting instead of the numeric solution of algebraic equations.

- Consider the two-loop five-point non-planar "double pentagon" integral:

$$\begin{gathered} D_1 = l_1^2, \quad D_2 = (l_1 - p_1)^2, \quad D_3 = (l_1 - p_1 - p_2)^2, \quad D_4 = l_2^2 \\ D_5 = (l_2 + p_4 + p_5)^2, \quad D_6 = (l_2 + p_5)^2, \quad D_7 = (l_1 - l_2)^2, \quad D_8 = (l_1 - l_2 + p_3)^2 \\ D_9 = (l_1 + p_5)^2, \quad D_{10} = (l_2 - p_1)^2 \quad D_{11} = (l_2 - p_1 - p_2)^2 \end{gathered} \tag{2}$$

  with $p_1^2 = p_2^2 = p_3^2 = p_4^2 = p_5^2 = 0$. Use either the Lee-Pameransky polynomial G or the Baikov polynomial P *on cut*, and run the critical point counting argument in *1308.6676* to find the number of master integrals on the sector $(1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0)$.

- (**Extra 5 points**) Use IBP reduction softwares, like LiteRed, FIRE or Kira to find the actual number of master integrals for the sector $(1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0)$ in the two above cases. Are they consistent with the argument in *1308.6676*? Please write down you conclusions and print your code for running these softwares.

  Hint: The IBP reduction may be very slow even if all s_{ij}'s are set to be integers. For LiteRed, you may try to solve the IBP for the specific sector only without going through all sectors. For FIRE or KIRA, you may set the reduction boundary by hand. Of course, if you have the access to a multi-CPU computer, the reduction is very easy, as long as you set all s_{ij}'s as integers.

2. (IBP reduction) Consider the two-loop four-point massless double box diagram:

$$D_1 = l_1^2, \quad D_2 = (l_1 - p_1)^2, \quad D_3 = (l_1 - p_1 - p_2)^2, \quad D_4 = (l_2 + p_1 + p_2)^2,$$
$$D_5 = (l_2 - p_4)^2, \quad D_6 = l_2^2, \quad D_7 = (l_1 + l_2)^2$$
$$D_8 = (l_1 + p_4)^2, \quad D_9 = (l_2 + p_1)^2 \tag{3}$$

with $p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$. We use the Mandelstam variables $s = 2p_1 \cdot p_2$ and $t = 2p_1 \cdot p_4$. Use softwares to reduce the integral $G[1,,1,1,1,1,1,1,-1,-1]$ to the linear combination of the following master integrals,

$$G[0,0,1,0,0,1,1,0,0], \quad G[0,1,0,0,1,0,1,0,0],$$
$$G[0,1,0,1,0,1,1,0,0], \quad G[1,0,1,1,0,1,0,0,0],$$
$$G[0,1,0,1,1,1,1,0,0], \quad G[0,1,1,0,1,1,1,0,0],$$
$$G[1,1,1,1,1,1,1,0,0], \quad G[1,1,1,1,1,1,1,-1,0] \tag{4}$$

Note that with LITERED you must run

FindSymmetries[***, EMs − > True];

first, to find the symmetries. (*** is name you set for this integral family). Otherwise you may not get 8 master integrals.

Hint: no matter which software you are using, the master integral list may not be exactly the same as (4). You need to reduce the target first, and then reduce my list (4) to the master integral list given in the software. Then a back substitute will give you the reduction needed for this exercise.